STABILIZATION OF TEMPERATURE AND PRESSURE OF A REAL GAS IN A STOPPED PIPE

B. L. Krivoshein, V. P. Radchenko, and M. G. Khublaryan

Inzhenerno-Fizicheskii Zhurnal, Vol. 13, No. 2, pp. 168–176, 1967

UDC 536.244:662.959.63

Nonisothermal unsteady flow of a real gas in a pipe is studied. A numerical method is given for solving a system of quasi-linear differential equations that describe the flow of a real gas in a stopped pipe. The calculated results are discussed.

1. Using the equations of conservation of mass, motion, and energy in the form of [1], the equation of state of a real gas in Berthelot form, and ignoring, as usual [1], the change in the geometric height and velocity of the gas along the coordinate and with respect to time, we obtain

$$\frac{\partial G}{\partial x}=\frac{a}{z_0T}\left(\frac{P}{T}\frac{\partial T}{\partial t}+\frac{P}{z_0}\frac{\partial z_0}{\partial t}-\frac{\partial P}{\partial t}\right),$$

$$\frac{\partial P}{\partial x}=-b\,\frac{z_0TG^2}{P},$$

$$\frac{\partial T}{\partial t}=-\frac{1}{a}\,\frac{z_0TG}{P}\,\frac{\partial T}{\partial x}-c_0\,\frac{z_0^2G^3T^4}{P^3}\,\frac{\partial z_0}{\partial T}+$$

$$+m\,\frac{T}{P}\left(z_0+T\,\frac{\partial z_0}{\partial T}\right)\frac{\partial P}{\partial t}+n\,\frac{z_0T\,(T_0-T)}{P},$$

$$a=f/R;\; b=\lambda R/2gDf^2;$$

$$c_0=\lambda AR^3/2gDf^3c_p;\; m=AR/c_p;$$

$$n=K\pi DR/c_pf;$$

$$z_0=1+\frac{9}{128}\,\frac{P}{P_c}\,\frac{T_c}{T}\left(1-6\,\frac{T_c^2}{T^2}\right). \qquad (1)$$

Under these assumptions, the problem boils down to finding the solution $P = P(x,t)$, $T = T(x,t)$, and $G = G(x, t)$ of system (1) in the domain D_0 ($0 \le x \le L$, $t \ge 0$), which satisfies the initial conditions

$$P(x,\ 0)=f(x);\; T(x,\ 0)=\varphi(x)\quad (0<x<L), \qquad (2)$$

and the boundary conditions

$$G(0,\ t)=0;\; G(L,\ t)=0. \qquad (3)$$

With this formulation, solution of system (1)–(3) in finite analytic form involves insurmountable difficulties. Therefore, a numerical method for solving this boundary-value problem on a computer is proposed below.

2. For convenience of computer solution, we shall transform system (1)–(3). If we eliminate $\partial G/\partial x$ from the first equation and solve the new system for the derivatives $\partial P/\partial t$ and $\partial T/\partial t$, and if we also use the dimensionless variables $P^0 = P/P_c$, $T^0 = T/T_c$, $G^0 = G/G_H$, $x^0 = x/L$, and $t^0 = tc/L$, we obtain

$$\frac{\partial P^0}{\partial t^0}=a_1z_0\left\{b_1\,\frac{1}{G^0}\,\frac{\partial^2P^0}{\partial (x^0)^2}-\right.$$

$$-b_2G^0\left(z_0+T^0\,\frac{\partial z_0}{\partial T^0}\right)\frac{\partial T^0}{\partial x^0}+$$

$$+b_3G^0\left[1-\frac{6}{(T^0)^2}\right]\frac{\partial P^0}{\partial x^0}-\left(z_0+T^0\,\frac{\partial z_0}{\partial T^0}\right)\times$$

$$\times\left[c_1(G^0)^3(T^0)^3(P^0)^{-2}z_0\,\frac{\partial z_0}{\partial T^0}-\right.$$

$$\left.\left.-n_1(T_0^0-T^0)\right]\right\}\Big/\left[1-m\left(z_0+T^0\,\frac{\partial z_0}{\partial T^0}\right)^2\right],$$

$$\frac{\partial T^0}{\partial t^0}=a_1z_0\,\frac{T^0}{P^0}\left\{b_1\,\frac{1}{G^0}\left(z_0+T^0\,\frac{\partial z_0}{\partial T^0}\right)\frac{\partial^2(P^0)^2}{\partial (x^0)^2}+\right.$$

$$+b_2G^0\left[m\left(z_0+T^0\,\frac{\partial z_0}{\partial T^0}\right)^2-2\right]\frac{\partial T^0}{\partial x^0}+$$

$$+b_3G^0\left[1-\frac{6}{(T^0)^2}\right]\left(z_0+T^0\,\frac{\partial z_0}{\partial T^0}\right)\frac{\partial P^0}{\partial x^0}-$$

$$-\left[c_1(G^0)^3(T^0)^3(P^0)^{-2}z_0\,\frac{\partial z_0}{\partial T^0}-\right.$$

$$\left.\left.-n_1(T_0^0-T^0)\right]\right\}\Big/\left[1-m\left(z_0+T^0\,\frac{\partial z_0}{\partial T^0}\right)^2\right],$$

$$\frac{\partial (P^0)^2}{\partial x^0}=-b_4z_0(G^0)^2T^0,$$

$$a_1=L/cP_c;\; b_1=mP_c^2/4abG_HL^2;\; b_2=G_HT_c/2aL;$$

$$b_3=9mG_HT_c/256aL;$$

$$b_4=2bLG_H^2T_c/P_c^2;\; c_1=c_0G_H^3;\; n_1=nT_c. \qquad (4)$$

The boundary conditions in the case in question obviously have the form

$$G^0(0,\ t^0)=G^0(1,\ t^0)\equiv 0. \qquad (5)$$

The intitial distribution P^0, T^0, and G^0($t^0 < 0$) is the solution of the system of equations obtained from (4) provided that the derivatives of the unknown functions with respect to time are zero:

$$G^0=1,$$

$$\frac{d(P^0)^2}{dx^0}=-b_4z_0T^0,$$

$$\frac{dT^0}{dx^0}=-c_2\,\frac{(T^0)^3z_0}{(P^0)^2}\,\frac{\partial z_0}{\partial T^0}+n_2(T_0^0-T^0),$$

$$x^0=0,\; P^0=P_H^0;\; T^0=T_H^0, \qquad (6)$$

where $c_2 = ac_0LG_H^2/T_c$ and $n_2 = anL/G_H$.

Let us replace the derivatives of system (4) at the nodes of the rectangular network by the difference relations

$$\left(\frac{\partial P^0}{\partial t^0}\right)_{i,k} \cong \frac{P^0_{i,k+1} - P^0_{i,k}}{\tau};$$

$$\left(\frac{\partial T^0}{\partial t^0}\right)_{i,k} \cong \frac{T^0_{i,k+1} - T^0_{i,k}}{\tau};$$

$$\left[\frac{\partial^2 (P^0)^2}{\partial (x^0)^2}\right]_{i,k} \cong \frac{(P^0)^2_{i-1,k} - 2(P^0)^2_{i,k} + (P^0)^2_{i+1,k}}{h^2};$$

$$\left(\frac{\partial P^0}{\partial x^0}\right)_{i,k} \cong \frac{P^0_{i+1,k} - P^0_{i,k}}{h};$$

$$\left[\frac{\partial (P^0)^2}{\partial x^0}\right]_{i,k} \cong \frac{(P^0)^2_{i+1,k} - (P^0)^2_{i,k}}{h}. \qquad (7)$$

By analogy with the linear case for hyperbolic equations, we replace $\partial T^0/\partial x^0$ at the node (i,k) by the difference relation $(T^0_{i,k+1} - T^0_{i,k})/h$ or $(T^0_{i,k} - T^0_{i-1,k})/h$, according to the sign coefficient of the derivative in (7) into the second equation of (4). If we substitute (7) into (4), we obtain a system of difference equations that approximate (4):

$$P^0_{i,k+1} = P^0_{i,k} \times$$

$$\times \left\{1 - \alpha B_1 \frac{(z_0)_{i,k}}{G^0_{i,k}\left[1 - m\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)^2_{i,k}\right]} \times \right.$$

$$\left. \times (P^0_{i+1,k} + 2P^0_{i,k} + P^0_{i-1,k})\right\} +$$

$$+ \alpha B_1 \frac{(z_0)_{i,k}}{G^0_{i,k}\left[1 - m\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)^2_{i,k}\right]} \times$$

$$\times [(P^0_{i+1,k} + P^0_{i,k}) P^0_{i+1,k} + (P^0_{i-1,k} + P^0_{i,k}) P^0_{i-1,k}] -$$

$$-\alpha h (z_0)_{i,k} \left\{B_2 G^0_{i,k}\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)_{i,k} (T^0_{i,k} - T^0_{i-1,k}) - \right.$$

$$\left. - B_3 G^0_{i,k}\left[1 - \frac{6}{(T^0)^2_{i,k}}\right](P^0_{i+1,k} - P^0_{i,k})\right\} \times$$

$$\times \left\{1 - m\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)^2_{i,k}\right\}^{-1} + \alpha h^2 \left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)_{i,k} \times$$

$$\times \left\{C_1 (G^0)^3_{i,k}\left[\frac{(T^0)^3 z_0}{(P^0)^2} \frac{\partial z_0}{\partial T^0}\right]_{i,k} - N_1 (T^0_0 - T^0_{i,k})\right\},$$

$$T^0_{i,k+1} = T^0_{i,k} \times$$

$$\times \left\{1 + \frac{\alpha B_1}{P^0_{i,k}} \frac{(z_0)_{i,k}\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)_{i,k}}{G^0_{i,k}\left[1 - m\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)_{i,k}\right]} \times \right.$$

$$\left. \times [(P^0)^2_{i+1,k} - 2(P^0)^2_{i,k} + (P^0)^2_{i-1,k}]\right\} +$$

$$+ \alpha h \frac{T^0_{i,k}}{P^0_{i,k}} (z_0)_{i,k} \left\{B_2 G^0_{i,k}\left[m\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)^2_{i,k} - 2\right] \times \right.$$

$$\times (T^0_{i,k} - T^0_{i-1,k}) + B_3 G^0_{i,k}\left[1 - \frac{6}{(T^0)^2_{i,k}}\right] \times$$

$$\left. \times (P^0_{i+1,k} - P^0_{i,k})\right\} - \alpha h^2 \left\{C_1 (G^0)^3_{i,k}\left[\frac{(T^0)^3 z_0}{(P^0)^2} \frac{\partial z_0}{\partial T^0}\right]_{i,k} - \right.$$

$$\left. - N_1 (T^0_0 - T^0_{i,k})\right\} \Big/ \left[1 - m\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)^2_{i,k}\right],$$

$$G^0_{i,k+1} = \frac{1}{2}\left(G^0_{i-\frac{1}{2},k+1} + G^0_{i+\frac{1}{2},k+1}\right),$$

$$G^0_{i-\frac{1}{2},k+1} = \mathrm{sign}(P^0_{i-1,k+1} - P^0_{i,k+1}) \times$$

$$\times \left[\frac{1}{b_4 (z_0 T^0)_{i-\frac{1}{2},k+1}} \left|\frac{(P^0)^2_{i,k+1} - (P^0)^2_{i-1,k+1}}{h}\right|\right]^{1/2},$$

$$G^0_{i+\frac{1}{2},k+1} = \mathrm{sign}(P^0_{i,k+1} - P^0_{i+1,k+1}) \times$$

$$\times \left[\frac{1}{b_4 (z_0 T^0)_{i+\frac{1}{2},k+1}} \left|\frac{(P^0)^2_{i+1,k+1} - (P^0)^2_{i,k+1}}{h}\right|\right]^{1/2},$$

$$(i = 1, 2, \ldots, n-1;\ k = 1, 2, \ldots), \qquad (8)$$

where n is the number of intervals the integration range $(0 \le x^0 \le 1)$ is divided into; and $\alpha = \tau/h^2$; $h = 1/n$; $B_1 = b_1/P_c$; $B_2 = b_2(t_0/P_c)$; $B_3 = b_3(L/cP_c)$; $C_1 = c_1(L/cP_c)$; $N_1 = n_1(L/cP_c)$.

The values $(z_0 T^0)_{i-1/2,k+1}$ and $(z_0 T^0)_{i+1/2,k+1}$ are found by linear interpolation between the values at adjacent modes. Formulas (8) approximate Eqs. (4) with accuracy to O(h).

Let us discuss approximation of the boundary conditions. As can be seen from Eqs. (5), all of the unknown functions at the boundary nodes, which are situated on the lines $x^0 = 0$ and $x^0 = 1$, cannot be determined from the boundary conditions. For the boundary nodes we must obtain difference relations that approximate the differential equations at the boundaries. We will assume that Eqs. (4) are also satisfied at the boundaries $x^0 = 0$ and $x^0 = 1$. If we eliminate $\partial^2 (P^0)^2/\partial (x^0)^2$ from (4), we obtain

$$\frac{\partial P^0}{\partial t^0} = \frac{P^0}{T^0} \frac{1}{m\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)} \frac{\partial T^0}{\partial t^0} = z_0 \times$$

$$\times \frac{B_2 G^0 \frac{\partial T^0}{\partial x^0} + C_1 (G^0)^3 (T^0)^3 (P^0)^{-2} z_0 \frac{\partial z_0}{\partial T^0} - N_1 (T^0_0 - T^0)}{m\left(z_0 + T^0 \frac{\partial z_0}{\partial T^0}\right)},$$

$$\frac{\partial (P^0)^2}{\partial x^0} = -b_4 z_0 T^0 (G^0)^2. \qquad (9)$$

If we approximate the second equation of (9) at the boundary nodes $(0, k+1)$ and $(n, k+1)$, we find

$$\frac{(P^0)^2_{1,k+1} - (P^0)^2_{0,k+1}}{h} = -b_4 [z_0 T^0 (G^0)^2]_{0,k+1},$$

$$\frac{(P^0)^2_{n,k+1} - (P^0)^2_{n-1,k+1}}{h} = -b_4 [z_0 T^0 (G^0)^2]_{n,k+1}. \qquad (10)$$

Here, the derivative $\partial (P^0)^2/\partial x^0$ is represented at the boundaries by one-sided differences. Considering (5), we obtain

$$P^0_{0,k+1} = P^0_{1,k+1};\ P^0_{n,k+1} = P^0_{n-1,k+1} \quad (k = 0, 1, 2, \ldots). \qquad (11)$$

Approximation of the first equation of (9) at the boundary nodes $(0, k)$ and (n, k) results in difference

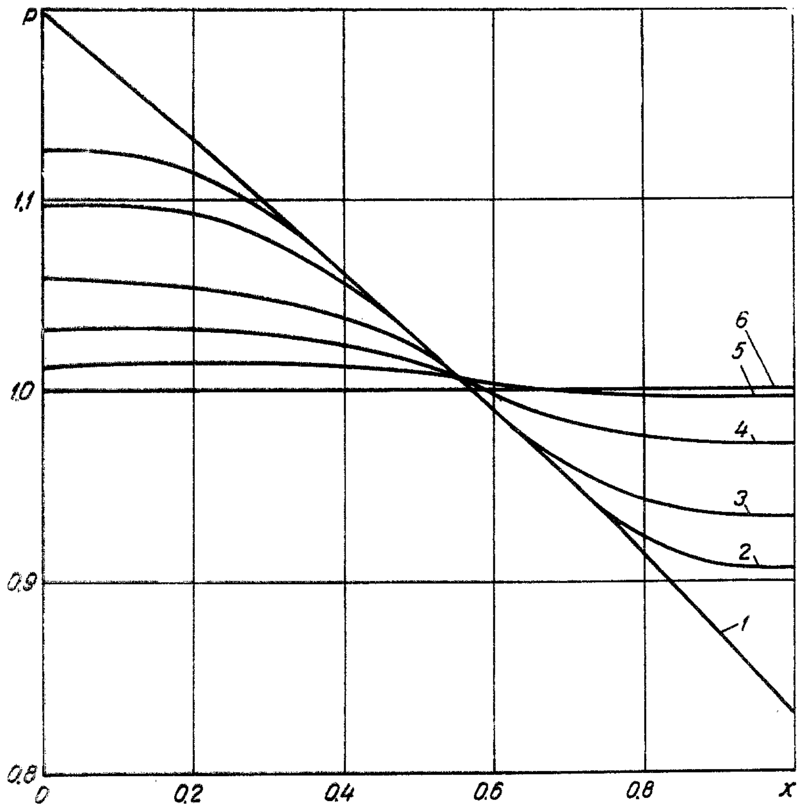

Fig. 1. Pressure stabilization curves: 1) $t^0 = 0$; 2) 0.25; 3) 0.5; 4) 1.0; 5) 1.5; 6) 2.0.

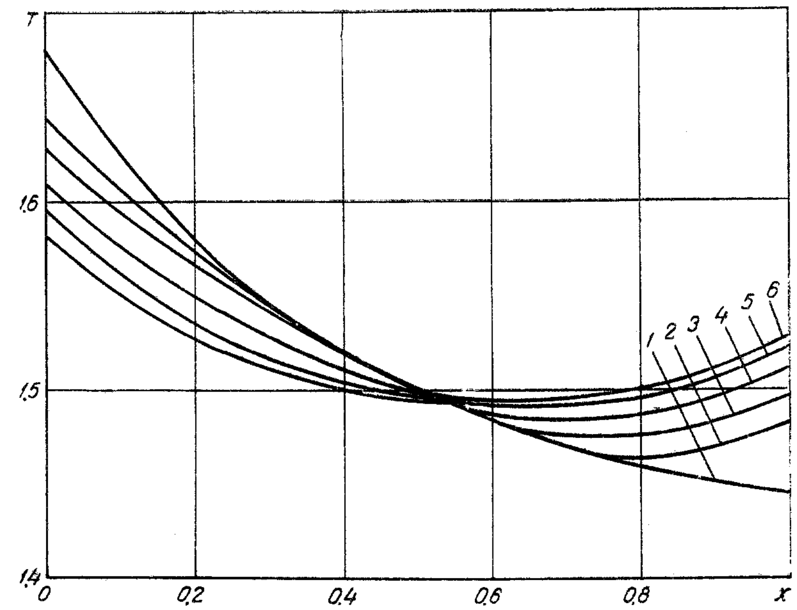

Fig. 2. Temperature stabilization curves: 1) $t^0 = 0$; 2) 0.25; 3) 0.5; 4) 1.0; 5) 1.5; 6) 2.0.

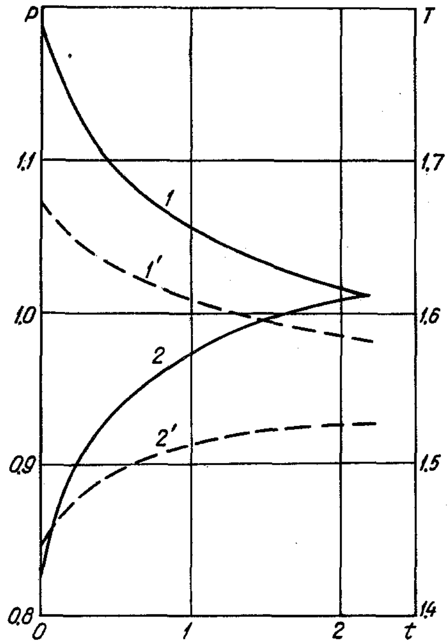

Fig. 3. Temperature and pressure at ends of pipe versus time: 1) $P^0(0)$; 1') $T^0(0)$; 2) $P^0(1)$; 2') $T^0(1)$.

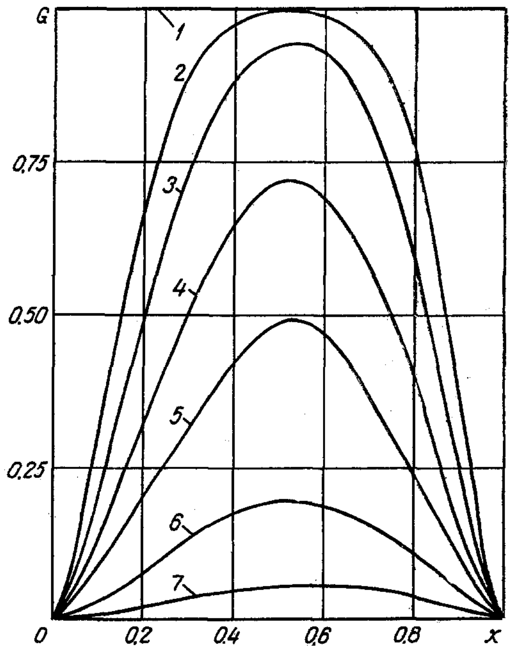

Fig. 4. Flow-rate stabilization curves: 1) $t^0 = 0$; 2) 0.25; 3) 0.5; 4) 1.0; 5) 1.5; 6) 2.0; 7) 2.25.

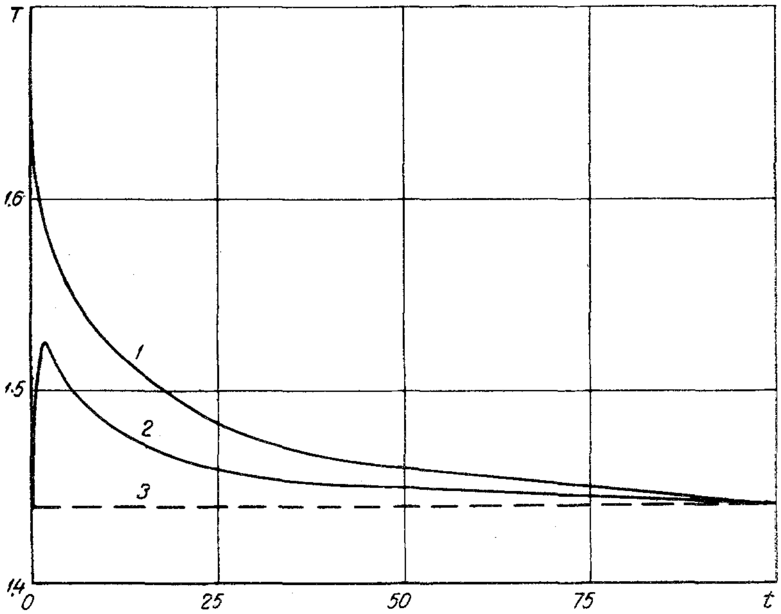

Fig. 5. Temperature at ends of pipe versus time: 1) $T^0(0)$; 2) $T^0(1)$; 3) T_0^0 .

relations from which we determine $T^0_{0,k+1}$ and $T^0_{n,k+1}$:

$$T^0_{0,k+1}=\frac{P^0_{0,k+1}-C}{B};\ T^0_{n,k+1}=\frac{P^0_{n,k+1}-C'}{B'},$$

$$B=\frac{P^0_{0,k}}{T^0_{0,k}}\ \frac{1}{m\left(z_0+T^0\dfrac{\partial z_0}{\partial T^0}\right)};$$

$$C=-\frac{\tau(z_0)_{0,k}N_1(T^0_0-T^0_{0,k})}{m\left(z_0+T^0\dfrac{\partial z_0}{\partial T^0}\right)}+$$

$$+P^0_{0,k}\left[1-\frac{1}{m\left(z_0+T^0\dfrac{\partial z_0}{\partial T^0}\right)_{0,k}}\right];$$

$$B'=\frac{P^0_{n,k}}{T^0_{n,k}}\ \frac{1}{m\left(z_0+T^0\dfrac{\partial z_0}{\partial T^0}\right)_{n,k}};$$

$$C'=-\frac{\tau(z_0)_{n,k}N_1(T^0_0-T^0_{n,k})}{m\left(z_0+T^0\dfrac{\partial z_0}{\partial T^0}\right)}+$$

$$+P^0_{n,k}\left[1-\frac{1}{m\left(z_0+T^0\dfrac{\partial z_0}{\partial T^0}\right)_{n,k}}\right]$$

$$(k=0,\ 1,\ 2,\ \ldots). \tag{12}$$

Relations (6) and (12), together with boundary conditions (5), represent a complete system of equations for determining the unknown functions at the boundary nodes on the lines $x^0=0$ and $x^0=1$. The order of error in this boundary-condition approximation is 0(h).

3. The explicit difference scheme (7), (11), and (12) was used in the numerical solution [6]. The program for the BESM-2M computer has two parts.

The first part, on the basis of a standard program, solves system (6) of ordinary differential equations by the Runge-Kutta method with a constant step h for finding the initial distribution $P^0(0, x^0)$ and $T^0(0, x^0)$.

The second part realizes calculation by difference scheme (7), (11), and (12). The program permits variations of the step h within the limits of the operational memory and operates with information that is completely storable in the operational memory without the use of magnetic drums and tapes, except for the cases provided for by the internal instructions of the computer or standard subroutines.

4. As an example, a calculation was made with the following initial data: L = 100 km, D = 0.7 m, $P_H = 55\cdot10^4$ N/m^2, G_H = 100 kg/sec, T_H = 320° K, and T_0 = 285° K. The gas was methane; $P_c = 45.8\cdot10^4$ N/m^2 and T_c = 190° K.

The initial conditions were ($t^0<0$): $G^0=1$, $x^0=0$, $T^0=1.67$, $P^0=1.2$ (for solving system (6)).

The boundary conditions were ($t^0>0$): $x^0=0$, $G^0=0$; $x^0=1$, $G^0=0$.

Figure 1 shows curves of pressure stabilization after stopping of the pipe. With time, at the left end ($x^0=0$) the pressure drops, owing to return flow from the higher-pressure region to the lower-pressure region. On the right end ($x^0=1$) the pressure increases, as compared with the steady value. It is interesting to note that at the point $x^0=0.55$ the pressure remains constant during stabilization and equals the steady value $P^0(0;\ 0.55)$.

An approximation formula for determining the length l^0 of the segment to the constant-pressure point during stabilization under isothermal conditions is given in [6]. As applied to our conditions, $l^0=0.55$.

With isothermal gas motion, the pressure remains constant at the point $x^0=0.50$ [4,5]. Thus, the formulas in [3,4] give results that differ by 5–10% from the numerical solution for nonisothermal gas motion. The position of the constant-pressure point is a function of the temperature distribution along the length of the pipe.

Figure 2 shows curves of the temperature depression along the pipe for different times. It is apparent from Fig. 2 that at the left end the gas temperature decreases with time due to the Joule-Thomson effect, since $P^0(0, t^0)$ becomes less than $P^0(0, 0)$. On the right end, $P^0(1, t^0)$ increases and, therefore, the real-gas temperature increases (up to the moment of pressure stabilization). Figure 3 shows curves of pressure(1, 2) and temperature (1', 2') versus time. When $t^0\geq 2$–3, the pressure-stabilization process is practically completed. During this time, the gas front, at the speed of sound, covers a distance of twice (or three times) the length of the pipe. The gas flow is practically stopped at $t^0\geq 2$–3.

Figure 4 shows curves of the mass flow-rate distribution along the pipe at various times. It can be seen that when $t^0\geq 2$–3, $G^0\to 0$, i.e., the gas flow is stopped for all practical purposes. The pressure-stabilization time for the conditions of our example is, as calculated by the approximation formula in [3], about 15 min. Numerical calculation gives a similar result of 10–12 min. Thus, the formula in [3] can be used for practical applications. After stopping of the gas, due to heat transfer with the ground, the temperature at the right and left ends begins to drop and when $t^0\to\infty$ it becomes equal to t^0 (Fig. 5). The temperature curve corresponding to the right end of the pipe ($x^0=1$) passes through a maximum at $t^0=2$. In practice, when $t^0\geq 100$ the gas temperature is stabilized, i.e., it reaches the ground temperature. After pressure stabilization, the last equation of system (1) takes the form

$$\frac{dT}{dt}=\frac{n(z_0)_m}{P_m}(T_0-T)T. \tag{13}$$

If we integrate (13), after reduction to dimensionless form we obtain

$$T^0=T^0_0\left\{1-\left(1-\frac{T^0_0}{T^0_1}\right)\exp\left[-\beta(t^0-t^0_1)\right]\right\}^{-1}, \tag{14}$$

where $\beta=K\pi DRz_0T_0L/c_p f P_m c$; t^0_1 is the time for pressure stabilization to P^0_m; and T^0_1 is the gas temperature at the moment of pressure stabilization.

If we solve (14) for K, we find

$$K = \frac{\beta_0}{t^0 - t_l^0} \ln \left[\left(1 - \frac{T_0^0}{T_l^0}\right) \Big/ \left(1 - \frac{T_0^0}{T^0}\right) \right], \qquad (15)$$

where $\beta_0 = c_p f P_m c / \pi DRz_0 T_0 L$.

The temperature distribution after pressure stabilization (Fig. 5) is calculated by formula (14).

If we measure the temperature variation during stabilization, we can, using formula (15), determine the mean value of the coefficient of heat transfer from the pipe to the ground (K).

NOTATION

G is the weight flow rate; P is the pressure; T is the temperature; f and D are the cross-sectional area and tube diameter, respectively; λ is the hydraulic resistance coefficient; R and c_p are the gas constant and isobaric heat capacity; c is the speed of sound in gas; A is the thermal equivalent of work; K is the heat transfer from gas to soil with Newton-law temperature distribution; z_0 is the compressibility coefficient (from Berthelot state equation [2]); P_c and T_c are the critical parameters of the real gas; t is the time; x is the coordinate; τ and h are the time interval and coordinate, respectively.

REFERENCES

1. I. A. Charnyi, Fundamentals of Gasdynamics [in Russian], GTTI, 1961.
2. M. P. Vukalovich and I. I. Novikov, Equation of State of a Real Gas [in Russian], GEI, Moscow and Leningrad, 1948.
3. I. E. Khodanovich, Analytic Fundamentals of the Design and Operation of Gas Mains [in Russian], GTTI, p. 112, 1963.
4. Z. T. Galiullin and V. I. Chernikin, Izv. VUZ. Neft i gaz, no. 12, 1960.
5. F. B. Abutalliev, Problems of Computer Mathematics [in Russian], Izd. AN UzSSR, Tashkent, 1963.
6. I. S. Berezin and N. P. Zhidkov, Calculation Methods, Vols. 1 and 2 [in Russian], Fizmatgiz, Moscow, 1959.

31 October 1966

Institute of Natural Gases, Moscow